Corrigendum to: Farzinnia, N., and K. F. McCardle (2010). Bayesian Updating with Confounded Signals. *Communications in Statistics—Theory and Methods*, 39:6, 956-972.

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Abstract

Farzinnia and McCardle (2010) consider the problem of estimating two unknown parameters from a sequence of confounded signals using Bayesian updating. They provide a set of equations that can characterize the evolution and asymptotic properties of the updated estimates. There are, however, mistakes in those equations. The purpose of this note is to provide the correct formulae.

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In Section 4.3 of the original paper [Farzinnia and McCardle (2010)], the expression on the right side of Equation (12) on page 966 is incorrect. As a result, all the equations that appear in Theorem 4.1 [Equations (17)-(21) on page 968], Theorem 4.2 [Equations (23)-(27) on page 969] and Section 4.4 [Equations (28), (29), (31) and (32) on page 970] are incorrect. The purpose of this note is to point out the source of these mistakes and provide the correct formulae.

To start, the correct expression of Equation (12) should be

$$\begin{aligned} h\left(\theta_X, \theta_Y \mid R_1\right) \propto \exp\Bigg\{\frac{-1}{2}\Bigg[& \theta_X^2\left(\frac{1}{\sigma^2}+\frac{1}{\left(1-\rho_0^2\right)\tau_{X_0}^2}\right) \\ & -2\theta_X\left(\frac{R_1}{\sigma^2}+\frac{\mu_{X_0}}{\left(1-\rho_0^2\right)\tau_{X_0}^2}-\frac{\rho_0\mu_{Y_0}}{\left(1-\rho_0^2\right)\tau_{X_0}\tau_{Y_0}}\right)+\theta_Y^2\left(\frac{1}{\sigma^2}+\frac{1}{\left(1-\rho_0^2\right)\tau_{Y_0}^2}\right) \\ & -2\theta_Y\left(\frac{R_1}{\sigma^2}+\frac{\mu_{Y_0}}{\left(1-\rho_0^2\right)\tau_{Y_0}^2}-\frac{\rho_0\mu_{X_0}}{\left(1-\rho_0^2\right)\tau_{X_0}\tau_{Y_0}}\right) \\ & -2\theta_X\theta_Y\left(-\frac{1}{\sigma^2}+\frac{\rho_0}{\left(1-\rho_0^2\right)\tau_{X_0}\tau_{Y_0}}\right)\Bigg]\Bigg\}. \end{aligned}$$

The last term, which corresponds to the coefficient e in Equation (13) of the original paper, is the main reason for the ensuing mistakes. In particular, it appears in $ac-e^2$, which is the denominator for the updated means and the updated variances in Theorem 4.1. The correct expression for $ac-e^2$ should be

$$\frac{1}{\sigma^2\left(1-\rho_0^2\right)}\left(\frac{1}{\tau_{Y_0}^2}+\frac{1}{\tau_{X_0}^2}+\frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right)+\frac{1}{\left(1-\rho_0^2\right)\tau_{X_0}^2\tau_{Y_0}^2}.$$

In Theorem 4.1, the correct expression for the updated variances [Equations (17) and (18)] should be

$$\tau_{X_1}^2 = \frac{\frac{1}{\sigma^2}+\frac{1}{(1-\rho_0^2)\tau_{Y_0}^2}}{\frac{1}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2}+\frac{1}{\tau_{X_0}^2}+\frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right)+\frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}},$$

$$\tau_{Y_1}^2 = \frac{\frac{1}{\sigma^2}+\frac{1}{(1-\rho_0^2)\tau_{X_0}^2}}{\frac{1}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2}+\frac{1}{\tau_{X_0}^2}+\frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right)+\frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}};$$

the correct expression for the updated correlation [Equation (19)] is

$$\rho_1 = \frac{-\frac{1}{\sigma^2}+\frac{\rho_0}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}}{\sqrt{\left(\frac{1}{\sigma^2}+\frac{1}{(1-\rho_0^2)\tau_{X_0}^2}\right)\left(\frac{1}{\sigma^2}+\frac{1}{(1-\rho_0^2)\tau_{Y_0}^2}\right)}};$$

and the correct expression for the updated means [Equations (20) and (21)] are

$$\mu_{X_1} = \frac{\left(\frac{R_1}{\sigma^2} + \frac{\mu_{X_0}}{(1-\rho_0^2)\tau_{X_0}^2} - \frac{\rho_0\mu_{Y_0}}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)\left(\frac{1}{\sigma^2} + \frac{1}{(1-\rho_0^2)\tau_{Y_0}^2}\right)}{\frac{1}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}} + \frac{\left(\frac{R_1}{\sigma^2} + \frac{\mu_{Y_0}}{(1-\rho_0^2)\tau_{Y_0}^2} - \frac{\rho_0\mu_{X_0}}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)\left(-\frac{1}{\sigma^2} + \frac{\rho_0}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)}{\frac{1}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}}$$

and

$$\mu_{Y_1} = \frac{\left(\frac{R_1}{\sigma^2} + \frac{\mu_{Y_0}}{(1-\rho_0^2)\tau_{Y_0}^2} - \frac{\rho_0\mu_{X_0}}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)\left(\frac{1}{\sigma^2} + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2}\right)}{\frac{1}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}} + \frac{\left(\frac{R_1}{\sigma^2} + \frac{\mu_{X_0}}{(1-\rho_0^2)\tau_{X_0}^2} - \frac{\rho_0\mu_{Y_0}}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)\left(-\frac{1}{\sigma^2} + \frac{\rho_0}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)}{\frac{1}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}}.$$

The same mistakes have been repeated in Theorem 4.2. The correct expression for the updated variances [Equations (23) and (24)] should be

$$\tau_{X_j}^2 = \frac{\frac{j}{\sigma^2} + \frac{1}{(1-\rho_0^2)\tau_{Y_0}^2}}{\frac{j}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}},$$

$$\tau_{Y_j}^2 = \frac{\frac{j}{\sigma^2} + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2}}{\frac{j}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}};$$

the correct expression for the updated correlation [Equation (25)] is

$$\rho_j = \frac{-\frac{j}{\sigma^2} + \frac{\rho_0}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}}{\sqrt{\left(\frac{j}{\sigma^2} + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2}\right)\left(\frac{j}{\sigma^2} + \frac{1}{(1-\rho_0^2)\tau_{Y_0}^2}\right)}};$$

and the correct expression for the updated means [Equations (26) and (27)] should be

$$\mu_{X_j} = \frac{\left(\frac{j\overline{R}_j}{\sigma^2} + \frac{\mu_{X_0}}{(1-\rho_0^2)\tau_{X_0}^2} - \frac{\rho_0\mu_{Y_0}}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)\left(\frac{j}{\sigma^2} + \frac{1}{(1-\rho_0^2)\tau_{Y_0}^2}\right)}{\frac{j}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}} + \frac{\left(\frac{j\overline{R}_j}{\sigma^2} + \frac{\mu_{Y_0}}{(1-\rho_0^2)\tau_{Y_0}^2} - \frac{\rho_0\mu_{X_0}}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)\left(-\frac{j}{\sigma^2} + \frac{\rho_0}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)}{\frac{j}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}}$$

and

$$\mu_{Y_j} = \frac{\left(\frac{j\overline{R}_j}{\sigma^2} + \frac{\mu_{Y_0}}{(1-\rho_0^2)\tau_{Y_0}^2} - \frac{\rho_0\mu_{X_0}}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)\left(\frac{j}{\sigma^2} + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2}\right)}{\frac{j}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}} + \frac{\left(\frac{j\overline{R}_j}{\sigma^2} + \frac{\mu_{X_0}}{(1-\rho_0^2)\tau_{X_0}^2} - \frac{\rho_0\mu_{Y_0}}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)\left(-\frac{j}{\sigma^2} + \frac{\rho_0}{(1-\rho_0^2)\tau_{X_0}\tau_{Y_0}}\right)}{\frac{j}{\sigma^2(1-\rho_0^2)}\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right) + \frac{1}{(1-\rho_0^2)\tau_{X_0}^2\tau_{Y_0}^2}}.$$

These equations are then used to derive the asymptotic value of the parameter estimates in Section 4.4. The correct expression for Equations (28) and (29) should be

$$\lim_{j\to\infty} \tau_{X_j}^2 = \frac{(1-\rho_0^2)}{\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right)},$$

$$\lim_{j\to\infty} \tau_{Y_j}^2 = \frac{(1-\rho_0^2)}{\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right)}.$$

Finally, the correct expressions for Equations (31) and (32) should be

$$\lim_{j\to\infty} \mu_{X_j} = \frac{\frac{(\theta_X^* + \theta_Y^*)}{\tau_{Y_0}}\left(\frac{1}{\tau_{Y_0}} + \frac{\rho_0}{\tau_{X_0}}\right) + \frac{\mu_{X_0}}{\tau_{X_0}}\left(\frac{1}{\tau_{X_0}} + \frac{\rho_0}{\tau_{Y_0}}\right) - \frac{\mu_{Y_0}}{\tau_{Y_0}}\left(\frac{1}{\tau_{Y_0}} + \frac{\rho_0}{\tau_{X_0}}\right)}{\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right)},$$

$$\lim_{j\to\infty} \mu_{Y_j} = \frac{\frac{(\theta_X^* + \theta_Y^*)}{\tau_{X_0}}\left(\frac{1}{\tau_{X_0}} + \frac{\rho_0}{\tau_{Y_0}}\right) + \frac{\mu_{Y_0}}{\tau_{Y_0}}\left(\frac{1}{\tau_{Y_0}} + \frac{\rho_0}{\tau_{X_0}}\right) - \frac{\mu_{X_0}}{\tau_{X_0}}\left(\frac{1}{\tau_{X_0}} + \frac{\rho_0}{\tau_{Y_0}}\right)}{\left(\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}\right)}.$$

The above equations have two intuitive implications that are not shown in the original paper. First, the sum of the two limits equals $(\theta_X^* + \theta_Y^*)$. This means after observing a sufficiently large number of signals, the firm will be able to identify the sum of the two unknown parameters. Second, the limit of μ_{X_j} is a weighted average of μ_{X_0} and $\left(\theta_X^* + \theta_Y^* - \mu_{Y_0}\right)$. Specifically,

$$\lim_{j\to\infty} \mu_{X_j} = \alpha\mu_{X_0} + (1-\alpha)\left(\theta_X^* + \theta_Y^* - \mu_{Y_0}\right),$$

where

$$\alpha = \frac{\frac{1}{\tau_{X_0}^2} + \frac{\rho_0}{\tau_{X_0}\tau_{Y_0}}}{\frac{1}{\tau_{Y_0}^2} + \frac{1}{\tau_{X_0}^2} + \frac{2\rho_0}{\tau_{X_0}\tau_{Y_0}}}.$$

Similarly, the limit of μ_{Y_j} can be expressed as

$$\lim_{j\to\infty} \mu_{Y_j} = (1-\alpha)\mu_{Y_0} + \alpha\left(\theta_X^* + \theta_Y^* - \mu_{X_0}\right).$$